HOW WELL CAN GRAVITY BE RECOVERED USING TOPEX AND GPS DATA?

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When Topex is launched in mid-1992 it will carry a high quality GPS receiver which will operate in concert with a worldwide network of precision GPS ground receivers. The data from these receivers can be used to recover new information about the earth's gravity field at longer wavelengths. We have developed software and algorithms which will allow this gravity field information to be recovered with much greater efficiency than with traditional techniques. The basis for these algorithms is the gravity bin formulation and related filtering techniques that exploit the repeat orbit of Topex and the sparse matrix structure of the problem. We have used this new software to evaluate the expected improvement in the gravity field using multiple ten-day arcs of GPS data from Topex.

INTRODUCTION

The 1992 launch of NASA's Ocean Topography Experiment satellite, Topex/Poseidon, with its experimental Global Positioning System (GPS) receiver, and a succession of similar GPS-equipped missions thereafter, will enable a marked improvement of current earth gravity models as a result of the comprehensive, multi-dimensional global tracking coverage provided by GPS. For the first time, high precision tracking data will be acquired continuously over the entire globe, including the vast ocean basins and extensive land masses in eastern Europe and Asia that are now inaccessible to western ground based tracking systems. To quantify the information contained in the Topex-GPS data set we have developed new processing techniques which we call the "gravity bin algorithm" (Refs. 1,2). Here we extend previously reported results (Refs. 1,2) to include multiple 10-day arcs coinciding with the Topex ground track repeat cycle.

Topex will carry a dual frequency microwave altimeter accurate to a few centimeters for measuring ocean topography. The scientific potential of this precise altimetry can be fully realized only if Topex geocentric altitude can be independently determined to a few centimeters (Ref. 3). With conventional dynamic tracking techniques, this requires a highly accurate gravity model. Under the current mission plan, the gravity model for Topex will be refined with laser ranging data, which will

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be the baseline tracking data type, collected in the early months of Topex flight. Topex will also carry an experimental GPS flight receiver, capable of observing six GPS satellites simultaneously, to demonstrate the capability of GPS carrier phase and P-code pseudorange for precise orbit determination. In previous studies we have determined that sub-decimeter orbit accuracy can be achieved with just two hours of GPS tracking data, provided that simultaneous measurements are also made at six well distributed ground sites (Refs. 3,4). The precision GPS data from Topex can also be valuable for refining the earth gravity model.

The common procedure for gravity field recovery is to solve simultaneously for a large number of coefficients (possibly thousands) of a spherical harmonic expansion (Refs. 4-6). This procedure is computationally demanding and is usually performed on powerful supercomputers. The gravity bin algorithm reduces the computational demand by reducing the effective number of simultaneously estimated parameters to a few hundred satellite positional adjustments, or gravity bins, relating to the gravity field along the ground track of a single orbit. Because the earth's gravity field is fixed and the satellite ground track repeats, the perturbations due to the gravity field experienced by the satellite repeat nearly perfectly for each repeat of a given track. Data from many repeats of the same track can thus be efficiently combined to recover a small set of gravity perturbations along that track; the perturbations along distinct ground tracks are recovered separately, then combined in a final fast transformation that yields a conventional global harmonic solution that is essentially identical to that produced with the conventional simultaneous estimation approach. In short, by exploiting the repeat ground track of the satellite we are able to partition the global solution geographically, achieving small batches of simultaneously estimated parameters which can then be efficiently combined.

Once the gravity bins are determined, finite differencing of neighboring bins is performed to compute local accelerations. With the data from all distinct tracks we can form a grid of accelerations covering the globe (or at least that portion of the globe overflown by the satellite). These accelerations give a measure of the gravity field information contained in the GPS measurements of Topex.

GRAVITY BIN ALGORITHM

The basic algorithm is described in Refs. 1 and 2. Gravity bins are 3-D positional deviations of the low earth orbiter at each measurement time point over a period of a few hours. The epoch state of the orbiter and other pertinent parameters are also simultaneously adjusted. Assuming no other dynamic errors, the gravity bins are related to current state by the linearized equation,

$$\mathbf{r}(.) = \frac{\partial \mathbf{r}}{\partial \mathbf{r}_0}\mathbf{r}_0 + \frac{\partial \mathbf{r}}{\partial \mathbf{v}_0}\mathbf{v}_0 + \delta(t) \tag{1}$$

where $\mathbf{r}(t)$ is the adjustment to the current position vector of the spacecraft, $\mathbf{r}_0$ and $\mathbf{v}_0$ are the adjustments to the position and velocity vectors at epoch, and $\delta(t)$ is the vector of 3-D gravity bin parameters at time t. In the estimation process, these local gravity bin parameters replace the spherical harmonic coefficients. The local gravity correction Δg can be computed from the solutions of gravity bin parameters δ by

$$\Delta g(t) = \ddot{\delta}(t) - \frac{\partial g}{\partial r}\,\delta(t) \tag{2}$$

where g is the acceleration due to the nominal gravity field. Since the spherical harmonics are orthogonal, the *global* gravity model expressed in terms of spherical harmonics can be constructed from the *local* gravity correction by a process similar to the Fourier transformation. Using the accelerations, Δg, and the orthogonality properties ignores some information and will not give exactly the same results as estimating the spherical harmonic functions directly. To get the same result, the bins (δ's) may be used as measurements on the spherical harmonic coefficients. A priori information about the gravity field is combined with the information from the bins after the conversion to spherical harmonics. Converting bins to spherical harmonics is inexpensive compared to measurement processing.

The number of gravity bin parameters required for one Topex orbit is about 150, which compares with a thousand or so spherical harmonic terms to which Topex is sensitive and which might be adjusted in a full gravity solution with an extended data arc. Thus, for each orbit we have a large reduction in the number of parameters treated simultaneously. The price we pay for this reduction is that we must account for the correlations between bins over multiple orbits for a long arc, say, ten days of data. (The Topex ground track repeats every ten days.) The correlation between bin parameters arises from parameters such as the Topex state and station locations which must be treated as common parameters over a long arc. Our algorithms account for the correlations, with very little extra cost compared to the data processing of each single rev of low earth orbiter data.

To detail the algorithm that accounts for the correlation of the bin parameters we assume 90 days of Topex tracking data as an example. Since Topex ground tracks repeat every ten days, we first break the data into 9 contiguous ten day arcs. Since each ten-day arc consists of 127 revs, we further break each ten day arc into 127 individual orbits (about 2 hrs of data). Now we classify parameters into several categories:

1. Nuisance parameters — parameters whose values are independent from 2-hour arc to 2-hour arc and whose values are not needed, e.g. clocks, carrier phase biases.
2. Common parameters — parameters that are common for the entire ten day arc, e.g. Topex epoch state. A new Topex epoch state is solved for on each 10 day arc.
3. Repeating parameters — These are the gravity bins. They are called repeating since the same names are used on each 2-hour arc. They are distinct parameters on each 2-hour arc, but then are common on multiple ten day arcs since the effects of gravity repeat every 10 days.

All individual 2-hour arcs are first processed using standard Square Root Information Filtering (SRIF) techniques (Ref. 7). These are then combined in an optimal way into one grand solution for the 90 days. The CPU time for filtering the 2-hour arcs entirely dominates the computing time (Ref. 2). We detail the steps below.

Removing Nuisance Parameters

The first step toward efficient combining of multiple filter solutions into one grand solution is to permute all the nuisance parameters to the left-hand corner of the 2-hour SRIF array R and remove them from R. This is demonstrated in the following.

Write **x** for the parameter estimates and **z** for the transformed residuals. Then the least-squares estimate of **x** is given by

$$\mathbf{x} = R^{-1}\,\mathbf{z} \tag{3}$$

with a covariance

$$P = R^{-1}\,R^{-t}. \tag{4}$$

Let $\mathbf{x} = (\,\mathbf{x}_n{}^t, \mathbf{x}_r{}^t\,)^t$ where $\mathbf{x}_n$ includes all nuisance parameters and $\mathbf{x}_r$ other parameters of interest. Thus R can be written as

$$R = \begin{pmatrix} A & B \\ 0 & C \end{pmatrix} \tag{5}$$

with A associated with the nuisance parameters and C other parameters of interest, and B their correlation. The inverse of R is

$$R^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}BC^{-1} \\ 0 & C^{-1} \end{pmatrix} \tag{6}$$

and the product $R^{-1}\,R^{-t}$ is

$$R^{-1}\,R^{-t} = \begin{pmatrix} A^{-1}A^{-t} + D\,D^t & D\,C^{-t} \\ C^{-1}D^t & C^{-1}C^{-t} \end{pmatrix} \tag{7}$$

where $D = -A^{-1}\,B\,C^{-1}$. Hence, the least-squares estimate of $\mathbf{x}_r$ is

$$\mathbf{x}_r = [\,0,\ C^{-1}\,]\,\mathbf{z} \tag{8}$$

with a covariance

$$P_r = C^{-1}\,C^{-t} \tag{9}$$

In other words, removing parameters which are associated with the left-hand corner of an upper triangular SRIF matrix does not affect the solutions and covariances of the remaining parameters.

For orbit determination of a low earth satellite using GPS measurements, a large number of the estimated parameters are nuisance parameters. Hence the removal of these nuisance parameters substantially reduces the size of the problem involved in the subsequent combining process.

Combining Multiple One-Rev Solutions into a Ten-Day Solution

With the nuisance parameters removed, the remaining parameters are either repeating or common parameters. All common parameters are arranged to reside to the right of all repeating parameters in the SRIF matrix. When combining multiple one-rev solutions over a complete 10-day repeat cycle, one needs to combine only the lower right corner, which now contains information associated with common parameters only. That is, a Householder transformation need only be applied to a string of *small* matrices, as shown in Fig. 1. The upper part of the SRIF matrix remains unchanged by the combining process. Since there are only a handful of common parameters, the combining process is fast. Therefore, a full 10-day repeat cycle of a filtering process containing tens of thousands of parameters (mostly gravity bins) can be carried out at a computational cost essentially the same as processing each one-rev arc with only a few hundred parameters.

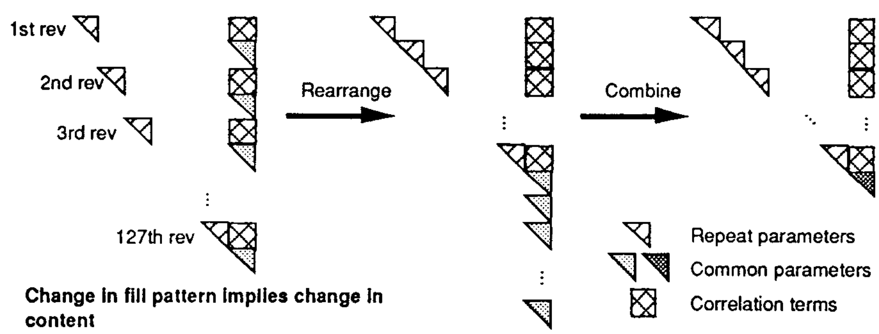

Fig. 1. Combining information of multiple one-rev arcs into a single 10-day arc

The rearrangement requires no computation and the combine step is accomplished with a sparse Householder transformation that takes advantage of the triangle structure of the common parameters. This transformation has sufficient vector length to take good advantage of vectorization on Cray type computers.

Combining Multiple Ten Day Solutions

In this second combining process, the gravity bin parameters are common to all 10-day arcs since they now represent the gravity effects on the Topex orbit over the same ground tracks. Topex initial epoch states are to be treated as (10-day) arc-dependent parameters to account for deviation from nominal orbits over different periods of time. Two combining steps are involved in this process, as shown in Fig. 2.

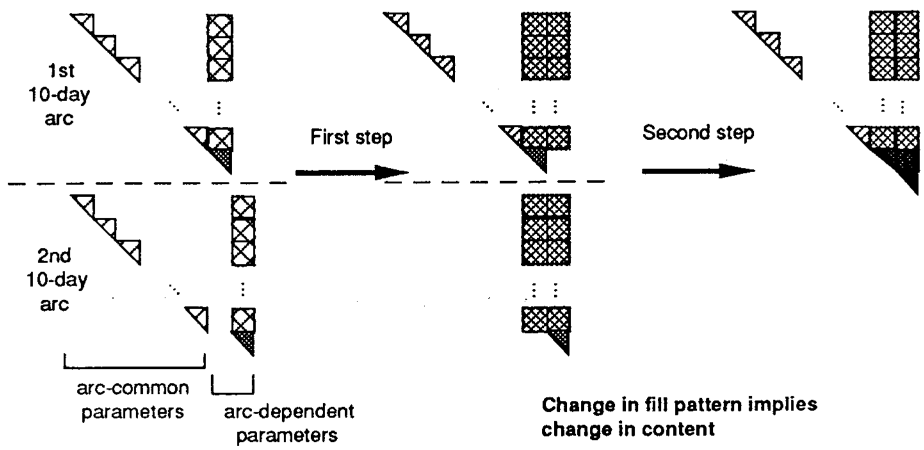

Fig. 2. Combining information of two 10-day arcs

The first step combines the information associated with the arc-common parameters (gravity bins) within each one-rev span from the first 10-day solution with those within the corresponding one-rev span from the second 10-day solution. Note that due to the block structure the rows containing *other* one-rev spans are not affected by this combining process. Hence the process is applied to a pair of one-rev spans at a time; it involves the zeroing of a triangular matrix corresponding to the bin parameters in the second 10-day solution and the transformation of the triangle corresponding to the bins in the first 10-day solution. The two rectangular matrices (which contain the correlation between the arc-common parameters and the arc-dependent parameters) are also transformed.

The second step combines all the transformed rectangular matrices in the second 10-day solution with the two (as yet unchanged) triangular matrices which contain the arc-dependent parameters, to form a single triangular matrix double in size. Since there are only a few arc-dependent parameters involved in this step, the combining process is very fast. To combine more than two 10-day arcs we simply repeat the above process by adding one new 10-day arc at a time to the combined multiple 10-day solutions. The first step will have the same number of operations while the second step will involve an increasing number of arc-dependent parameters; however, this will still be less than 250 parameters for 90 days of data.

Inversion of Combined SRIF Matrix

The result of the above combining process is an upper triangular SRIF matrix. The quantities of interest are the least-squares estimates $\hat{\mathbf{x}} = R^{-1}\,\mathbf{z}$ and its covariance $P = R^{-1}\,R^{-t}$ where R, $\hat{\mathbf{x}}$ and $\mathbf{z}$ now refer to all parameters involved in the combined solution. To extract these quantities, the SRIF matrix must be inverted and postmultiplied by its transpose. Directly inverting a matrix with a dimension of the order of 10^4 is prohibitively time consuming. This of course is not actually necessary, as a dominant portion (containing the gravity bins) of the combined SRIF matrix R has a block diagonal structure; each block of R can be separately inverted. The inverted matrix preserves the same block diagonal structure:

$$R^{-1} = \begin{pmatrix} B_1 & & & D_1 \\ & B_2 & & D_2 \\ & & \ddots & \vdots \\ & & B_N & D_N \\ & & & C \end{pmatrix}^{-1} = \begin{pmatrix} B_1^{-1} & & & -B_1^{-1} D_1 C^{-1} \\ & B_2^{-1} & & -B_2^{-1} D_2 C^{-1} \\ & & \ddots & \vdots \\ & & B_N^{-1} & -B_N^{-1} D_N C^{-1} \\ & & & C^{-1} \end{pmatrix}$$

where $B_1, B_2, \ldots, B_N$ are triangular matrices, each associated with the gravity bins in a one-rev span; C is also a triangular matrix associated with the arc-dependent parameters and the D's are rectangular matrices. Blank spaces denote null matrices. Therefore, the inversion of the huge combined SRIF matrix is reduced into inversions of small triangular matrices $B_1, B_2, \ldots, B_N$ and C, and the problem is greatly simplified.

COVARIANCE ANALYSIS

A covariance analysis was performed for the problem of determining gravity bins using GPS measurements made onboard Topex and at six globally located ground tracking sites, as shown in Fig. 3. These include the three NASA Deep Space Network (DSN) tracking sites at Goldstone, California; Madrid, Spain; and Canberra, Australia; and three other sites in Japan, Brazil and South Africa. A six-orbit-plane, 18-satellite constellation was assumed for GPS. Carrier phase and P-code pseudorange data were simulated at a rate of 50 samples per Topex revolution. At this sampling rate the gravity bins resolution is comparable to that of a 36x36 spherical harmonic expansion of the global field. Data over 10 days were used. Table 1 lists all the parameters estimated in the filter process, together with their a priori uncertainties. Although tight a priori uncertainties were assumed for station location and GPS orbits, which could be attained with ground tracking of GPS (Ref. 8), the results are in fact insensitive to these values. The a priori information for nuisance and repeating parameters is applied in *all* one-rev filter runs but to only one of these runs for common parameters. The Orbit Analysis and SImulation Software (OASIS) (Ref. 9) was used for the filtering process of the 127 one-rev data arcs. The nuisance parameters were then removed after permuting them to the top of the list. Only the repeating parameters (the gravity bins) and the common parameters were saved and arranged in that order. The 127 one-rev solutions were then combined, adding one at a time, to give a 10-day solution. Multiple 10-day arc solutions were formed by combining the first 10-day arc solution with itself. This approximation introduces only a small error since GPS has such uniform coverage and the Topex ground track repeats.

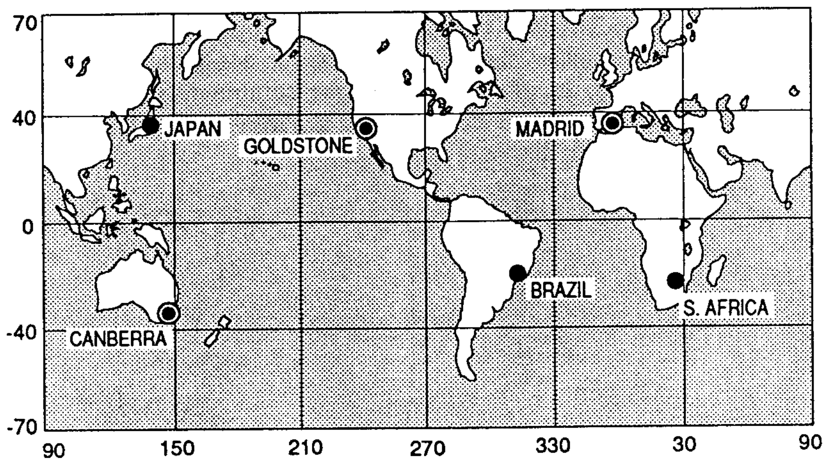

Fig. 3. A global tracking network

TABLE 1. ESTIMATED PARAMETERS AND A PRIORI UNCERTAINTIES

	Parameter Name	a priori σ	# of Parameters
Data:	P-code pseudorange	5 cm	
	carrier phase	0.5 cm	
	Data Rate	50 samples/rev	
	Cut-off Elevation	10 degrees	
Nuisance Parameters:	Carrier Phase Bias	10 km	144
	Clock Bias (white noise)	3 μsec	24
	GPS Epoch State	1 m; 0.1 mm/sec	108
	Zenith Troposphere (random walk)	20 cm bias; 1.3 cm batch to batch	6
Repeat Parameters:	Gravity Bins	1 km	150/rev
Common Parameters:	Station Location	5 cm each component	18
	Topex Epoch State	1 km; 1 m/sec	6

Results

In Ref. 2 it was shown that the sigma for acceleration due to gravity (Δg, Eq. 2) determined by the gravity bins was essentially independent of arc length up to 10 days. This can be explained as follows. Although longer data arcs will improve the estimation of the common parameters (in this case Topex epoch states and station location) their effects on gravity bins are highly correlated from one time to the next.

Since the gravity adjustment Δg is calculated from the second time difference of bin parameters, it is insensitive to such common parameters. Hence, improving these parameters by combining longer data arcs does not help improve gravity recovery accuracy. However, combining multiple 10-day repeat cycles will improve the accuracy. Fig. 4 shows the RMS error for the three acceleration components for arc 63 after combining 10, 20, and 40 days of data. The results scale as expected by approximately $\sqrt{n}$, where n is the number of ten day arcs. Extrapolating the result to 9 ten day arcs yields an average RMS error of .04 mgals (1 mgal = 1×10^{-3} cm/sec^2).

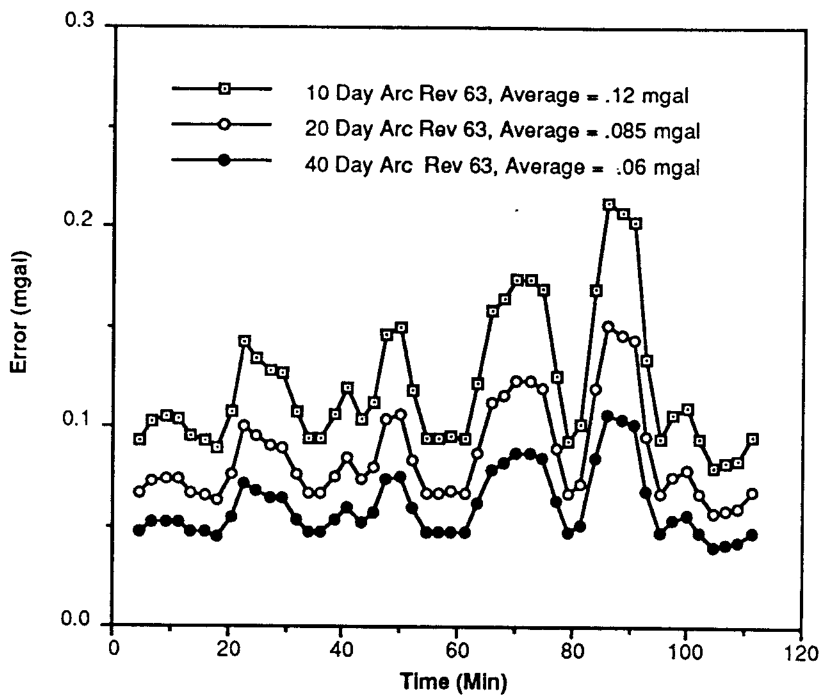

Fig. 4 Predicted RMS Error Arc 63

Arc 63 was chosen as typical; gravity sigmas vary slightly due to geometry from arc to arc. Fig. 5 shows the geometric variation in predicted error for arc 1 and arc 63. Note that although GPS coverage is very uniform there are only six ground stations. We believe it is the non-uniform distribution in ground coverage that causes the geometric variation.

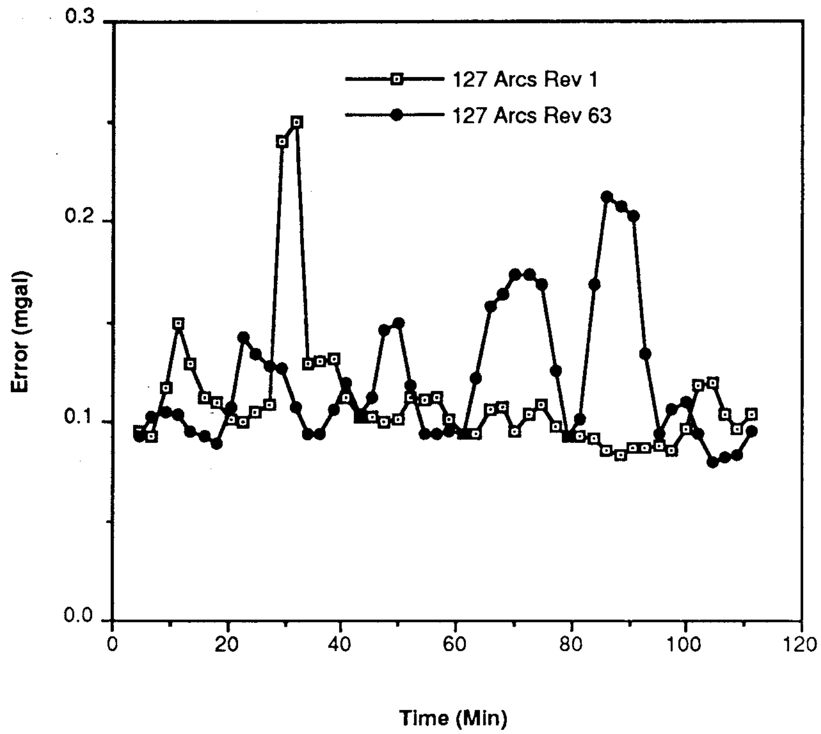

Fig. 5 Predicted RMS Error Arc 1 and 63

It would be interesting to compare the error predicted for the gravity bin technique with the error estimates contained in the covariance matrices of the current high precision gravity fields such as Goddard's GEM-T2 (Ref. 10). Software which will read the GEM-T2 covariance and compute the error in the acceleration at any point in a satellite orbit is currently being completed and results of the comparison will be given in a later report. The study by Marsh, et al. (Ref. 10) reports a predicted contribution to Topex altitude error of 12 cm from the GEM-T2 gravity model used in a conventional dynamic orbit solution. With a large ensemble of repeat arcs, the bin algorithm is projected to reduce Topex altitude error to well under 10 cm (Refs. 1,2) and shows promise for efficiently refining the gravity field for the Topex orbit.

Note that the process of averaging errors on the bins has limitations. Eventually the errors are limited by such things as non-gravitational force model errors on Topex, Earth Orientation errors, and station location errors. Future work will consider some of these error sources.

CONCLUSIONS

We have shown that the accelerations due to gravity may be recovered with GPS data from Topex by means of a highly efficient new filtering techniques. The recovered accelerations are estimated to be accurate to an average RMS value of .04

mgals after processing nine 10-day arcs. The errors are essentially uniform over the portion of the globe overflown by Topex.

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